Gauge-invariance in hadronic scattering processes[*]

C J Bomhof[†]

Department of Physics and Astronomy, Vrije Universiteit Amsterdam,

De Boelelaan 1081, NL-1081 HV Amsterdam, The Netherlands

High energy hadronic scattering processes can be described in terms of partonic scattering processes and parton distribution and fragmentation functions. These are bilocal matrix elements of quark operators. Colour gauge-invariance requires inclusion of a gauge link, which is characterized by its path. For transverse momentum dependent distribution and fragmentation functions, appearing in single spin asymmetries, different paths appear depending on the process. The difference is related to a time-reversal odd gluonic pole matrix element. We discuss briefly the path structure of the gauge link in more complicated processes.

The description of hadronic scattering processes is complicated by several aspects. Hadrons are not elementary particles, but rather bound states of quarks and gluons. The structure of hadrons in terms of the partons and the processes that bind them together are, up to today, mostly unknown. Obviously, these are all inherently non-perturbative properties of QCD and cannot be calculated using the method of Feynman diagrams. A very intuitive first approach to the treatment of hadronic scattering processes is provided by Feynman's parton model (Feynman 1969). The basic assumption of the parton model is that, on the time scale of the hard scattering process, we can regard the partons as essentially free particles. This picture has proven to be a very good first approximation in deep inelastic scattering (DIS: $\ell + h \to \ell' + X$), the simplest of all hadronic scattering processes. Schematically, the scattering cross section of DIS in the parton model can be written as $d\sigma^{h\ell} \propto \sum_q \int dx \ f^q(x) \, d\hat{\sigma}^{q\ell}(x)$. Here $d\hat{\sigma}^{q\ell}$ is the partonic scattering cross section that can be calculated perturbatively and the $f^q(x)$ are the distribution functions of parton species q with momentum fraction x. These distribution functions contain all the non-perturbative information of the hadrons. Hence they cannot (at present) be calculated from first principles. However, they can be measured in certain types of experiments and, then, be used to make predictions about other hadronic scattering processes.

At leading twist there are three different quark-distribution functions: the *unpolarised distribution* $f_1^q(x) = q(x)$ which measures the distribution of unpolarised quarks of flavour q in an unpolarised hadron, the *longitudinal polarisation distribution* or *helicity distribution* $g_1^q(x) = \Delta q(x)$ which measures the distribution of longitudinally polarised quarks in a longitudinally polarised hadron, and the *transverse polarisation distribution* or *transversity distribution* $h_1^q(x) = \delta q(x)$ which measures the distribution of transversely polarised quarks in a transversely polarised hadron. The functions f_1^q and g_1^q can be measured relatively easy in DIS. Measurements of g_1^q revealed that the quark helicities only account for about 20% of the helicity of the hadron. This 'spin crisis' sparked renewed interest in the role of quark-spin in hadrons and the transversity function h_1^q. Transversity, though, has proven to be much more difficult to measure than the other two functions. Consequently our knowledge of this function has fallen behind a little bit with our knowledge of the unpolarised and longitudinally polarised distribution functions. A more complete discussion on these issues can be found in (Ryckbosch 2004).

For a theoretical description of the distribution functions we need a proper definition of these objects. An appropriate framework is provided by the quark-correlator (Soper 1977) $\Phi_{ij}(k) =$

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[*]Contributed talk presented at the I3HP Topical Workshop at the University of St Andrews, Scotland, August 30 - September 1, 2004.

[†]*E-mail address*: cbomhof@nat.vu.nl

$(2\pi)^{-4} \int \mathrm{d}^4\xi \; e^{ik\cdot\xi} \; \langle P, S | \overline{\psi}_j(0) \, \mathcal{U}(0,\xi) \, \psi_i(\xi) \, | P, S \rangle$. All the distribution functions are projections of the quark-correlator: $f_1(x) = \int \mathrm{d}k^- \mathrm{d}^2\boldsymbol{k}_T \operatorname{Tr}\left[\gamma^+ \Phi(k)\right]$, $S_L \, g_1(x) = \int \mathrm{d}k^- \mathrm{d}^2\boldsymbol{k}_T \operatorname{Tr}\left[\gamma^+ \gamma_5 \Phi(k)\right]$ and $S_T^i \, h_1(x) = \int \mathrm{d}k^- \mathrm{d}^2\boldsymbol{k}_T \operatorname{Tr}\left[i\sigma^{i+} \Phi(k)\right]$. The operator $\mathcal{U}(0,\xi)$ is the gauge link, a path-ordered exponential. The gauge link is necessary to obtain a colour gauge-invariant definition of the quark-correlator. Colour gauge-invariance of the quark-correlator is a prerequisite for the distribution functions that are defined via the quark-correlator to be gauge-invariant and have any physical relevance.

As stated above, the gauge link $\mathcal{U}(0,\xi) \equiv \mathcal{P} \exp ig \int_C \mathrm{d}z \cdot A(z)$ is a path-ordered exponential connecting the fields at 0 and ξ along a certain path C. This has important implications for processes where the intrinsic transverse momenta of the partons play a role. If that is the case, *i.e.*, if one is sensitive to transverse directions, the points 0 and ξ in the definition of the quark-correlator will also be separated in the transverse direction ξ_T. If $\xi_T = 0$ the gauge link runs along the light-cone, if $\xi_T \neq 0$ it must include a transverse piece (Brodsky *et al.* 2002, Belitsky *et al.* 2003), leading to different possible integration paths. The integration path C that we have to take in a certain process is determined by the hard scattering part of that process. In fact, it can explicitly be calculated by taking all interactions of the current quark with the spectator partons into account (Boer and Mulders 2000). For example, in semi-inclusive deep inelastic scattering (SIDIS: $\ell + h_1 \rightarrow \ell' + h_2 + X$) the points 0 and ξ are connected to each other via light-cone future-infinity. In Drell-Yan scattering (DY: $h_1 + \overline{h}_2 \rightarrow \ell + \overline{\ell} + X$), however, they are connected via light-cone past-infinity. Due to the different gauge links in SIDIS and DY, the difference of the average transverse momentum of the quarks in the incoming proton in these two processes does not necessarily have to vanish. Instead, it is proportional to an object that is called a *gluonic pole* (Boer *et al.* 2003). The gluonic poles play a role in single spin asymmetries measured in experiment.

Recent studies have shown that scattering processes with more complicated partonic subprocesses than SIDIS and DY in general also have more complicated link structures (Bomhof *et al.* 2004). One new effect is an extra winding of the integration path C around light-cone future and past-infinity before connecting the points 0 and ξ. Another is the occurrence of traced Wilson loops in the quark-correlator. All these new structures occur in hadronic pion production processes, *e.g.* $p^\uparrow p \rightarrow \pi X$ and $\overline{p}^\uparrow p \rightarrow \pi X$. Such processes have been studied *e.g.* at FERMILAB and RHIC. The exact form of the gauge links in these processes is now under investigation. The newly found link structures will lead to enhancements of the gluonic poles, but there is still work to be done in finding out the specific observables in which one sees their effects. Other topics for future research are the implications that the new link structures have on issues such as universality and factorisation (Collins and Metz 2004).

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